Maxwell's Equation [In differential form] :-

$\text{Div } D = \rho$ — Gauss law of Electrostatic

$\text{Div } B = 0$ — Magnet

$\text{Curl } H = J + J_d$ — Modific^n in Ampere law

$\text{Curl } E = \frac{-\delta B}{\delta t}$ — Faraday law

Maxwell gives four eq^ns in which two is based upon electrostatics and two is based upon magnatostatics and the whole Maxwell's eq^n can be written as :-

(i) $\text{Div } D = \rho$
(ii) $\text{Div } B = 0$
(iii) $\text{Curl } H = J + J_d$
(iv) $\text{Curl } E = -\frac{\delta B}{\delta t}$

Maxwell's first eqn :-

1. This is also known as Gauss law of electrostatics. According to this law the divergence of electric displacement vector (D) is discontinuous i.e; it is equal to volume charge density (ρ).

$$\text{Div } D = \rho$$

where D is the electric displacement vector whose unit is coulomb/m^2

proof:- When a dielectric substance is placed in an electric field then two charges are created in the dielectric substance known as Bound charge and free charge.

From Gauss law of electrostatics, we h...

$S \to V$ Gauss (div)
$L \to S$ Stokes (curl)

$$\int E \cdot ds = \frac{1}{\epsilon_0}(q + q')$$

The bound charge is associated with bound charge density and free charge is associated with free charge density, which can be expressed as

$$\left\{ \begin{array}{l} q = \text{free charge} \\ q' = \text{bound charge} \end{array} \right.$$

$$\int E \cdot ds = \frac{1}{\epsilon_0}(q + q') \quad \text{—— (i)}$$

$$q = \int \rho \, dV \quad \text{—— (ii)}$$

$$q' = \int \rho' \, dV \quad \text{—— (iii)}$$

Now, from eq^n (i), (ii) & (iii), we get

$$\int E \cdot ds = \frac{1}{\epsilon_0}\left[\int \rho \, dV + \int \rho' \, dV\right] \quad \text{—— (iv)}$$

Now the bound charge density in term of polarisation can be expressed as :-

$$\rho' = -\,\mathrm{div}\, P = -\nabla \cdot P$$

then put in eq^n (4), it will

converts in the form

$$\int E \cdot ds = \frac{1}{\epsilon_0}\int \rho\, dV - \frac{1}{\epsilon_0}\int Div\, P\, dV$$

Now, from Gauss divergence theorem the surface integral can be converted into volume integral i.e;

$$\int Div\, E \cdot dV = \frac{1}{\epsilon_0}\int \rho\, dV - \frac{1}{\epsilon_0}\int Div\, P\, dV$$

$$\int Div\, E\, dV + \frac{1}{\epsilon_0}\int Div\, P\, dV = \frac{1}{\epsilon_0}\int \rho\, dV$$

$$\int Div\left(E + \frac{P}{\epsilon_0}\right) dV = \frac{1}{\epsilon_0}\int \rho\, dV$$

$$\frac{1}{\epsilon_0}\int Div\,(\epsilon_0 E + P)\, dV = \frac{1}{\epsilon_0}\int \rho\, dV$$

$$\int Div\, D\, dV = \int \rho\, dV$$

$$\boxed{Div\, D = \rho}$$

This is the required expression of Maxwell's First eqn which shows that divergence of electric displacement vector is discontinuous.